GRAVITATIONAL ENCOUNTERS IN THE RELATIVISTIC REGIME

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Characteristic Radii

Gravitational radius: $r_g = \dfrac{GM_\bullet}{c^2}$

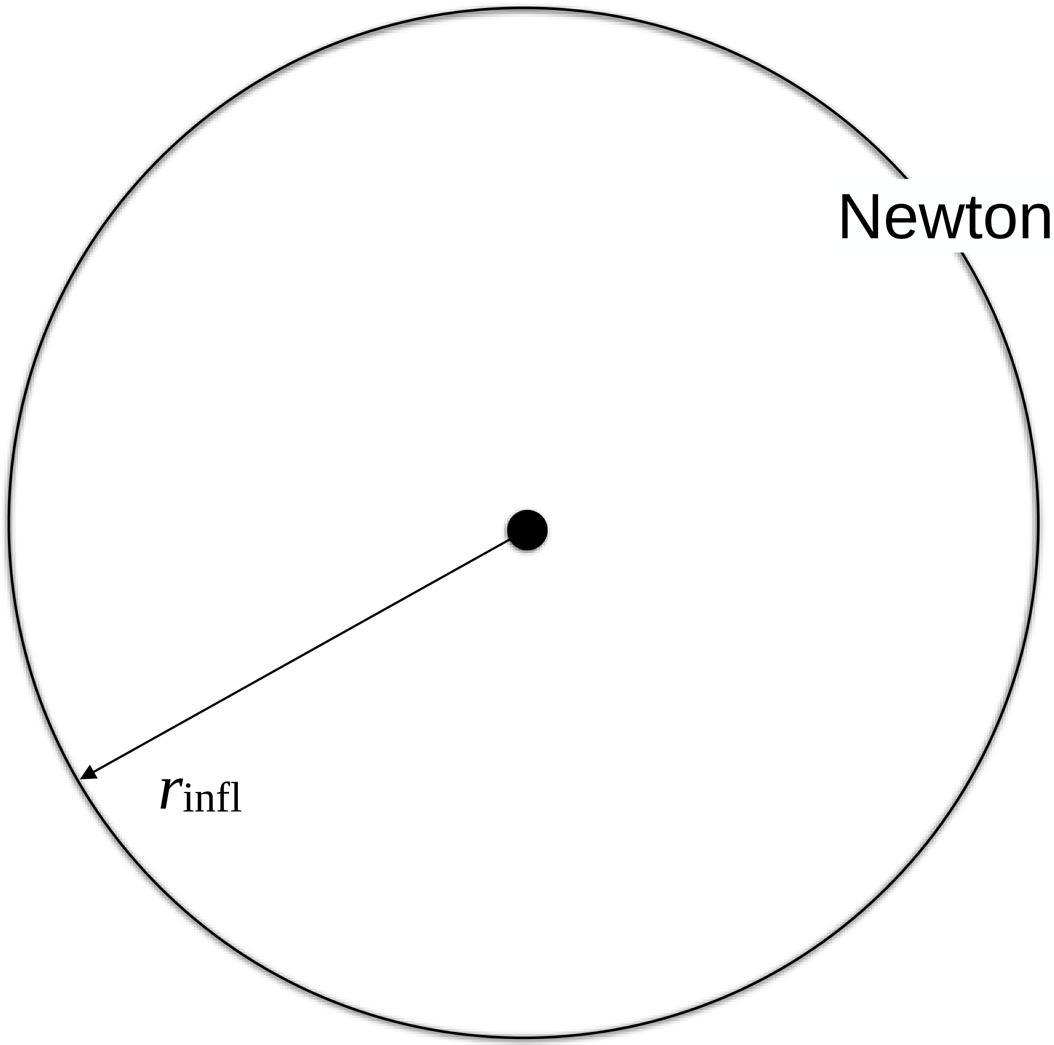
$$\approx 5 \times 10^{-8} \left(\frac{M_\bullet}{10^6 M_\odot} \right) \text{pc}$$

(gravitational) Influence radius:

$$r_{\text{infl}} = \frac{GM_\bullet}{\sigma^2}$$

$$\approx 0.5 \left(\frac{M_\bullet}{10^6 M_\odot} \right) \left(\frac{\sigma}{100 \text{ km s}^{-1}} \right)^{-2} \text{pc}$$

$$\boxed{\begin{aligned} \frac{r_g}{r_{\text{infl}}} &= \left(\frac{\sigma}{c} \right)^2 \\ &\approx 10^{-7} \left(\frac{\sigma}{100 \text{ km s}^{-1}} \right)^2 \end{aligned}}$$



Newton
r_{infl}

Gravitational encounters in the “Newton” regime:

!

- orbits are nearly linear

- the number of stars, N, is large

- encounters are random

(Eddington, Chandrasekhar, Ogorodnikov, Henon, Spitzer, Binney & Tremaine….)

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Fokker-Planck Equation for an Inverse-Square Force*

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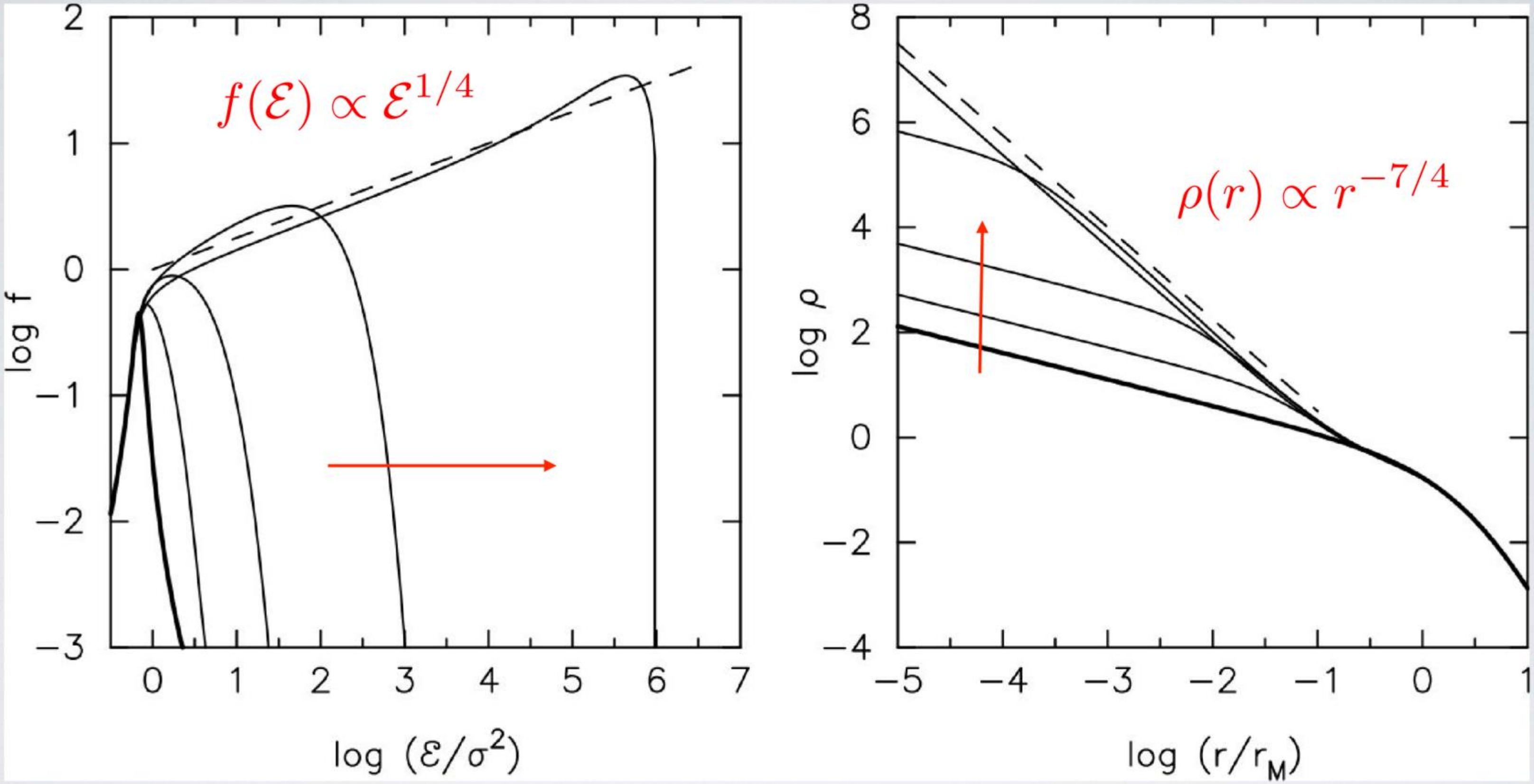
(Received August 31, 1956)

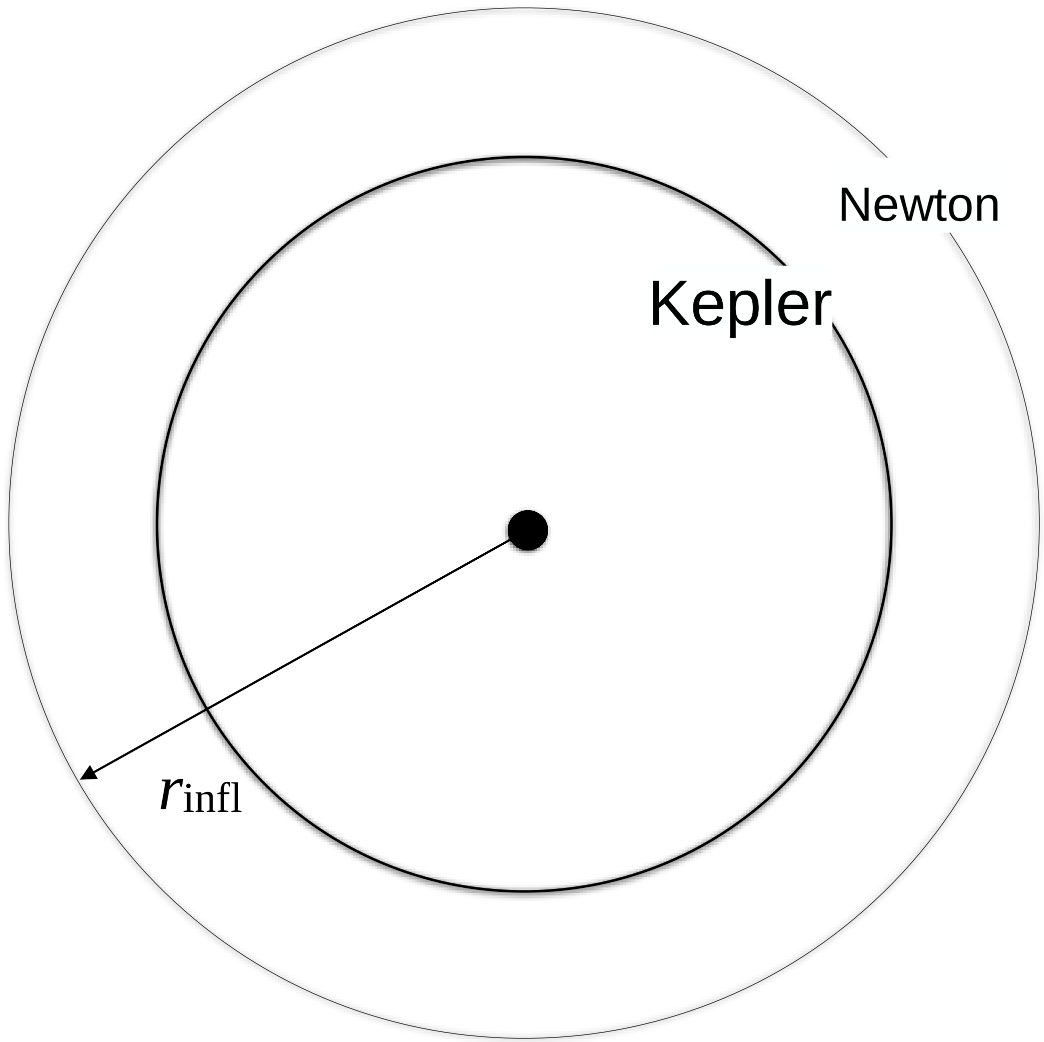
The contribution to the Fokker-Planck equation for the distribution function for gases, due to particle-particle interactions in which the fundamental two-body force obeys an inverse square law, is investigated. The coefficients in the equation, $\langle \Delta \mathbf{v} \rangle$ (the average change in velocity in a short time) and $\langle \Delta \mathbf{v} \Delta \mathbf{v} \rangle$, are obtained in terms of two fundamental integrals which are dependent on the distribution function itself. The transformation of the equation to polar coordinates in a case of axial symmetry is carried out. By expanding the distribution function in Legendre functions of the angle, the equation is cast into the form of an infinite set of one-dimensional coupled nonlinear integro-differential equations. If the distribution function is approximated by a finite series, the resultant Fokker-Planck equations may be treated numerically using a computing machine. Keeping only one or two terms in the series corresponds to the approximations of Chandrasekhar, and Cohen, Spitzer and McRoutly, respectively.

I. INTRODUCTION

IN dealing with the nonequilibrium properties of systems whose particles obey an inverse-square law of interaction, it is convenient to make use of the fact that under most circumstances small-angle collisions are much more important than collisions resulting in large momentum changes.[1] This leads to the method often used for treating such systems, in which one expands the collision integrand of the Boltzmann are two-body interactions described by the associated differential scattering cross sections, and (b) that the distribution function is isotropic in space and velocity space. Spitzer and collaborators[3,4] have extended this calculation to the case in which the distribution function is of the form $f^{(0)}+\mu f^{(1)}$, where $f^{(0)}$ and $f^{(1)}$ are isotropic and μ is the direction cosine between the particle trajectory and some preferred direction in space, and $f^{(1)}$ is assumed to be small.

Evolution to steady state (Newton)




Newton
Kepler
r_{infl}

Characteristics of the “Kepler” regime:

!

- orbits are nearly Keplerian

- the number of stars, N, is not large

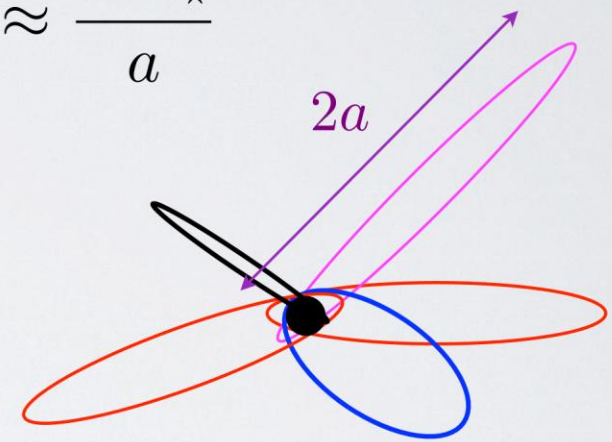
- encounters are correlated

Orbits are nearly Keplerian

→ **orbital orientations are (nearly) fixed**

→ **torques between stars are (nearly) fixed**

$$\frac{\text{torque}}{\text{orbit}} = |\langle \boldsymbol{F} \times \boldsymbol{r} \rangle| \approx \frac{Gm_\star}{a^2} \times a \approx \frac{Gm_\star}{a}$$



$$\begin{aligned} \text{net torque} &\approx \sqrt{N}\frac{Gm_\star}{a} \\ &\approx \frac{dL}{dt} \\ &\approx \frac{d}{dt}\sqrt{GM_\bullet a(1-e^2)} \\ &\approx \sqrt{GM_\bullet a}\,\frac{d}{dt}\sqrt{1-e^2} \end{aligned}$$

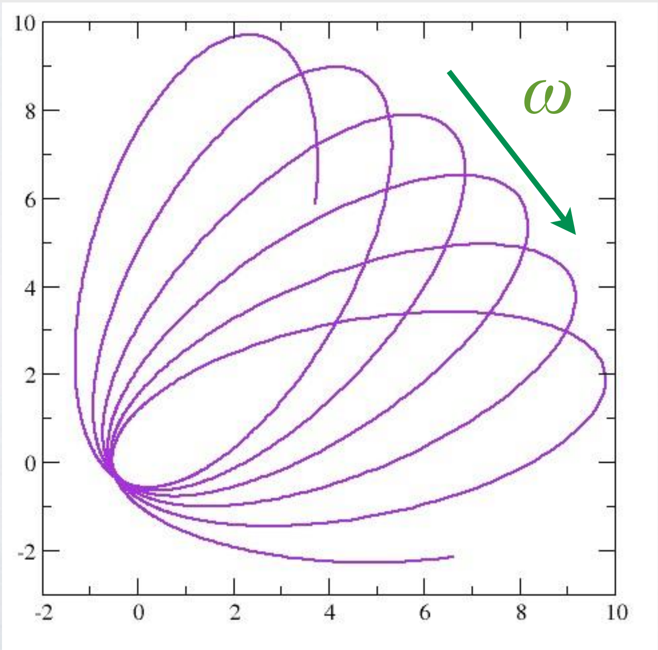
(Rauch & Tremaine 1996)

Coherence Time

Torques remain fixed for a time ~ t_{coh}, the coherence time.

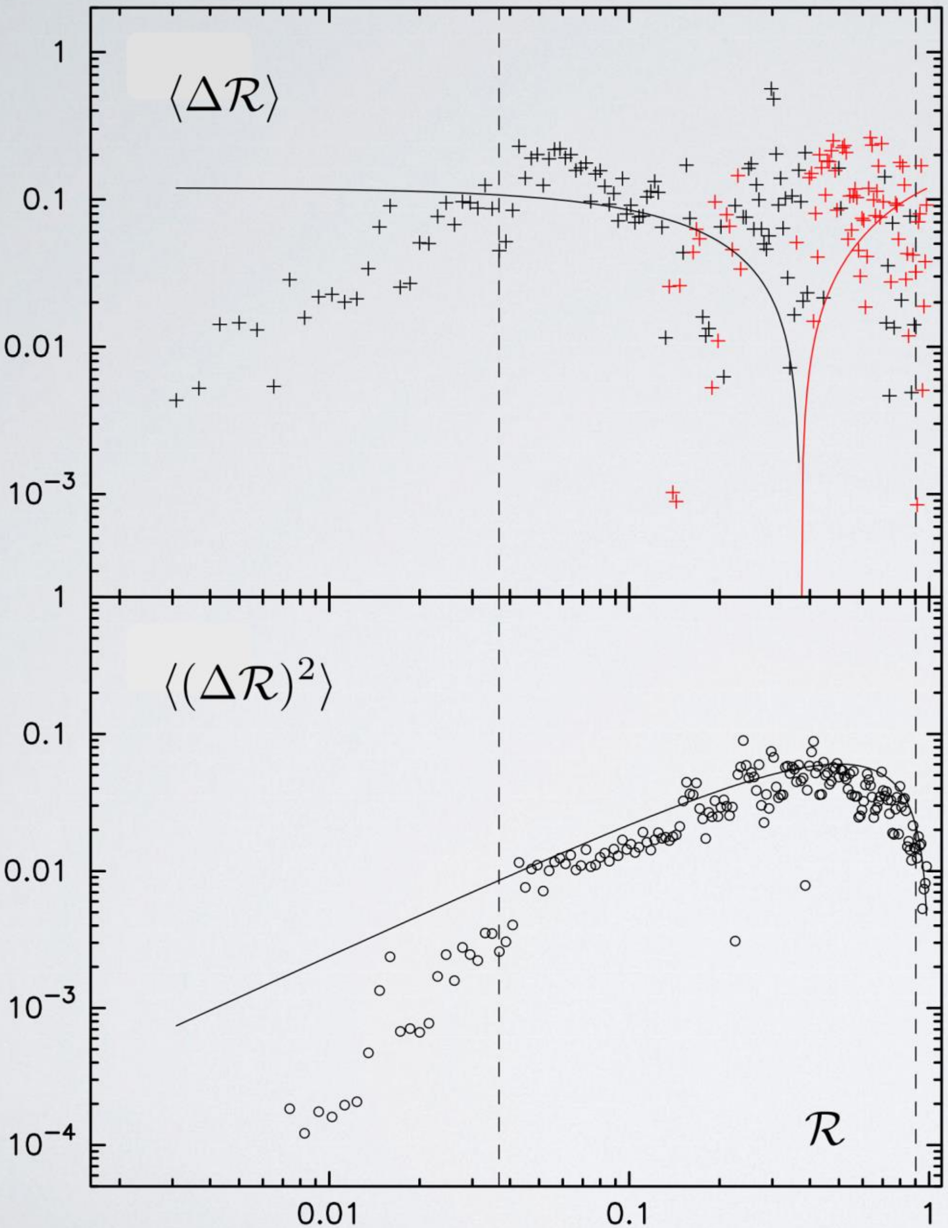
!

$t_{coh} \equiv$ the *typical* precessional time-scale



Apsidal Precession

ω = "argument of periastron"

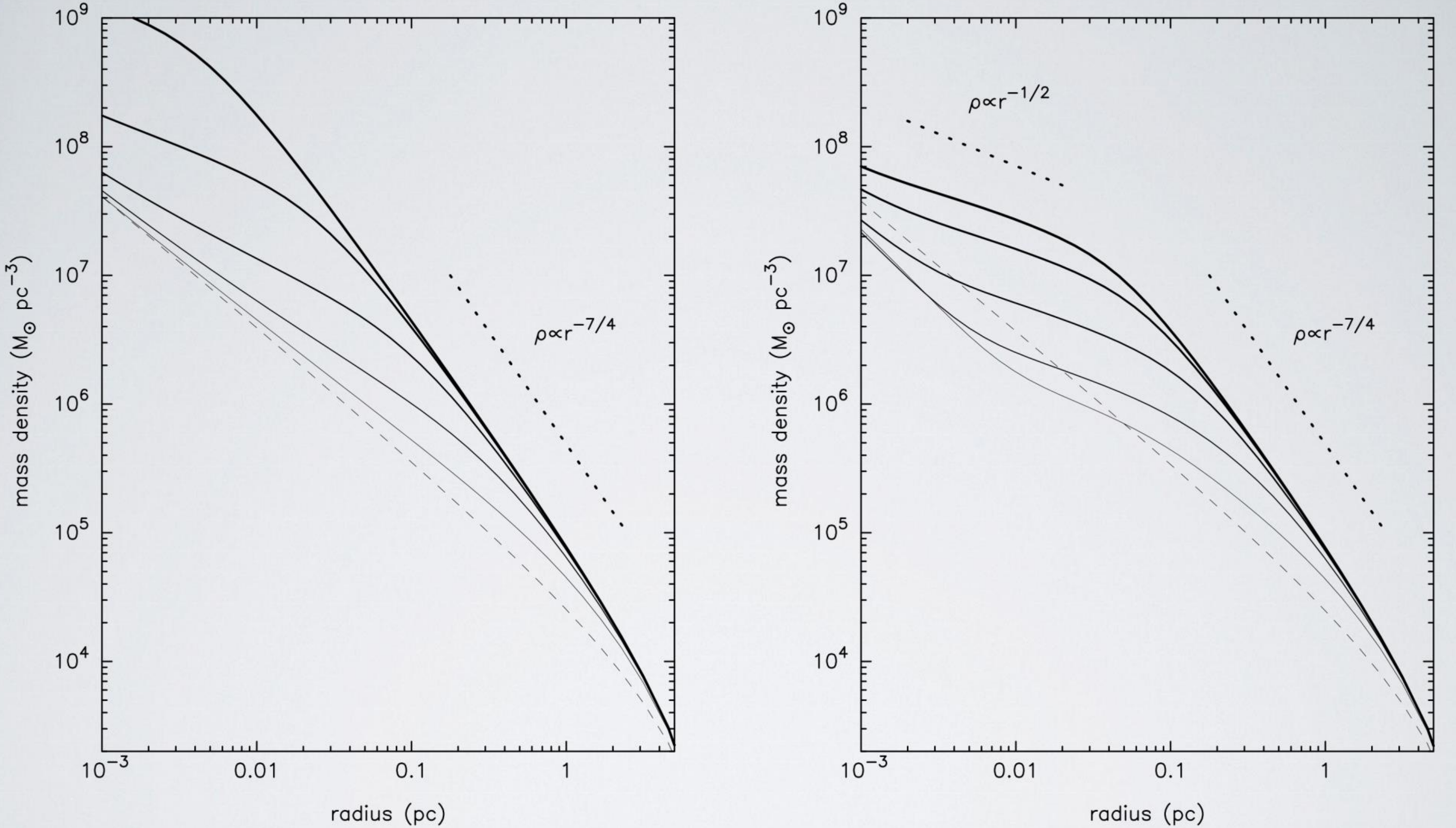


$$\mathcal{R} \equiv \frac{L^2}{L_c^2} \qquad \mathcal{E} = \frac{GM_\bullet}{2a}$$

$$\langle \Delta\mathcal{R} \rangle \approx 2\ A(\mathcal{E})\ (1 - 2\mathcal{R})$$

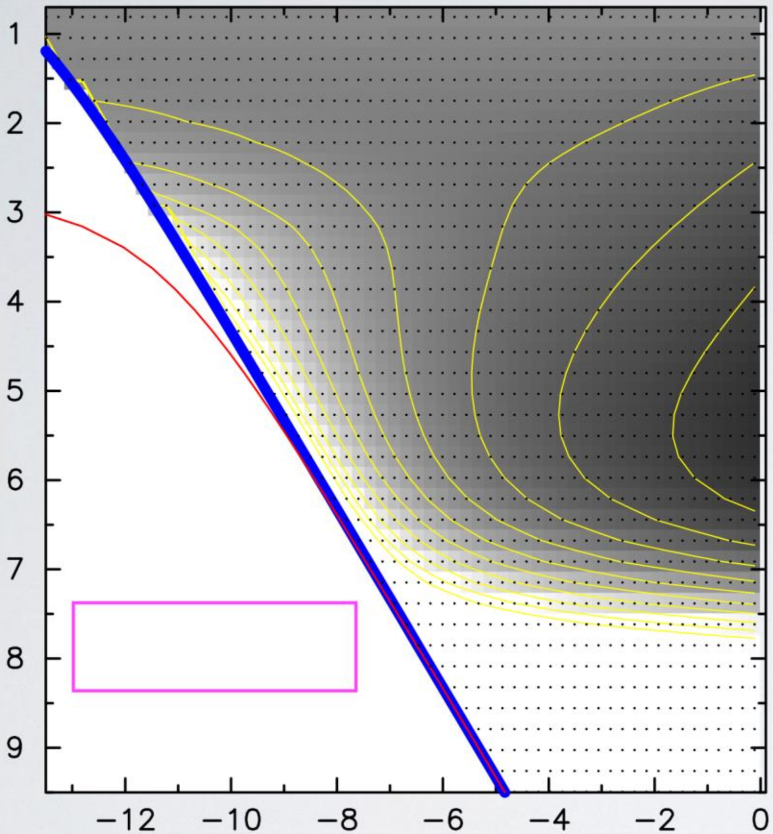
$$\langle (\Delta\mathcal{R})^2 \rangle \approx 4\ A(\mathcal{E})\ \mathcal{R}\,(1 - \mathcal{R})$$

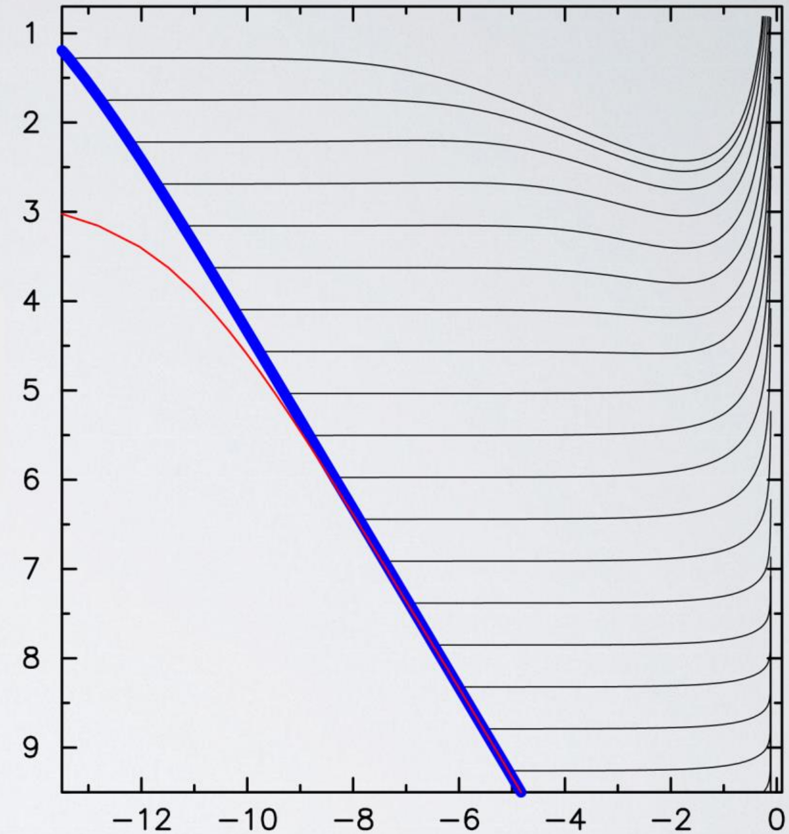
$$A(a) = \alpha^2 \left(\frac{M_\star(a)}{M_\bullet} \right)^2 \frac{1}{N(a)} \frac{t_{\mathrm{coh}}(a)}{P(a)^2}$$



10^9
10^8
10^7
10^6
10^5
10^4
mass density ($M_\odot$ pc^{-3})
$\rho \propto r^{-7/4}$
10^{-3}
0.01
0.1
1
radius (pc)
10^9
10^8
10^7
10^6
10^5
10^4
mass density ($M_\odot$ pc^{-3})
$\rho \propto r^{-1/2}$
$\rho \propto r^{-7/4}$
10^{-3}
0.01
0.1
1
radius (pc)

$$f = f(E, L, t)$$





$E \uparrow$

$L \rightarrow$

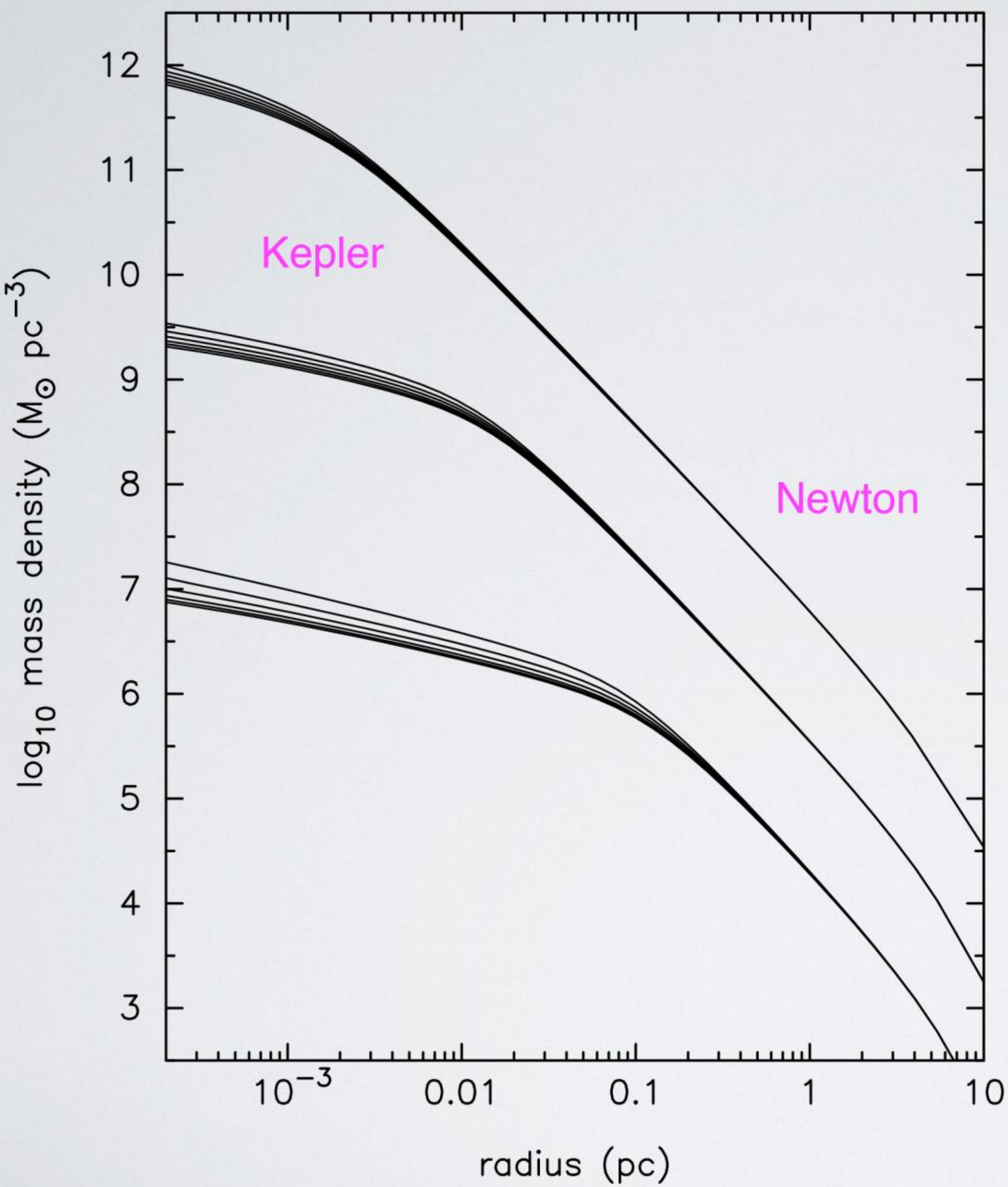
$f(E, L)$

Core radius:

$$\left|\frac{dL}{dt}\right|_{\text{Newton}} \approx \left|\frac{dL}{dt}\right|_{\text{Kepler}}$$

$$5\,\frac{G^2 m_\star M_\bullet \ln\Lambda}{(GM_\bullet)^{7/4}\, r_{\text{infl}}^{5/4}} \left(\frac{GM_\bullet}{2a}\right)^{1/4} \approx \frac{m_\star}{M_\bullet}\sqrt{\frac{GM_\bullet}{a^3}}$$

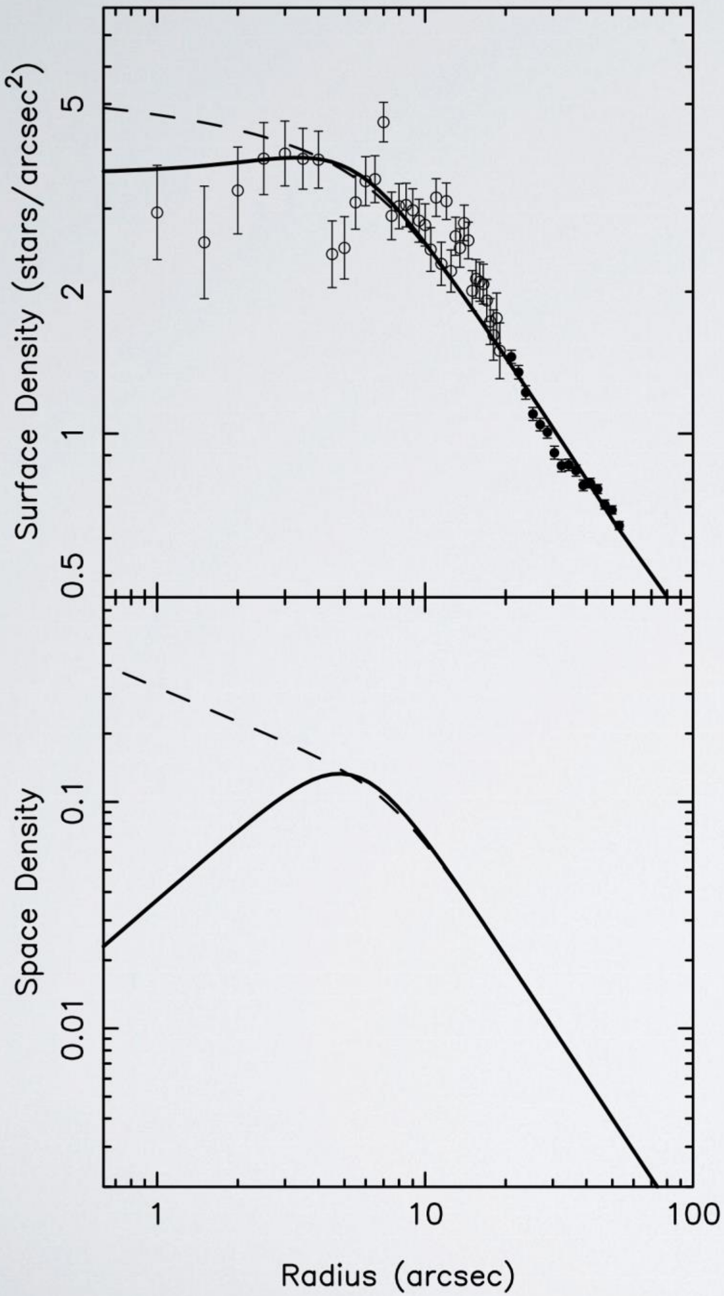
$$\boxed{\frac{a}{r_{\text{infl}}} \approx 0.03\left(\frac{\ln\Lambda}{15}\right)^{-4/5}.}$$



Kepler
Newton
log10 mass density (M⊙ pc−3)
radius (pc)
12
11
10
9
8
7
6
5
4
3
10−3
0.01
0.1
1
10

ρ

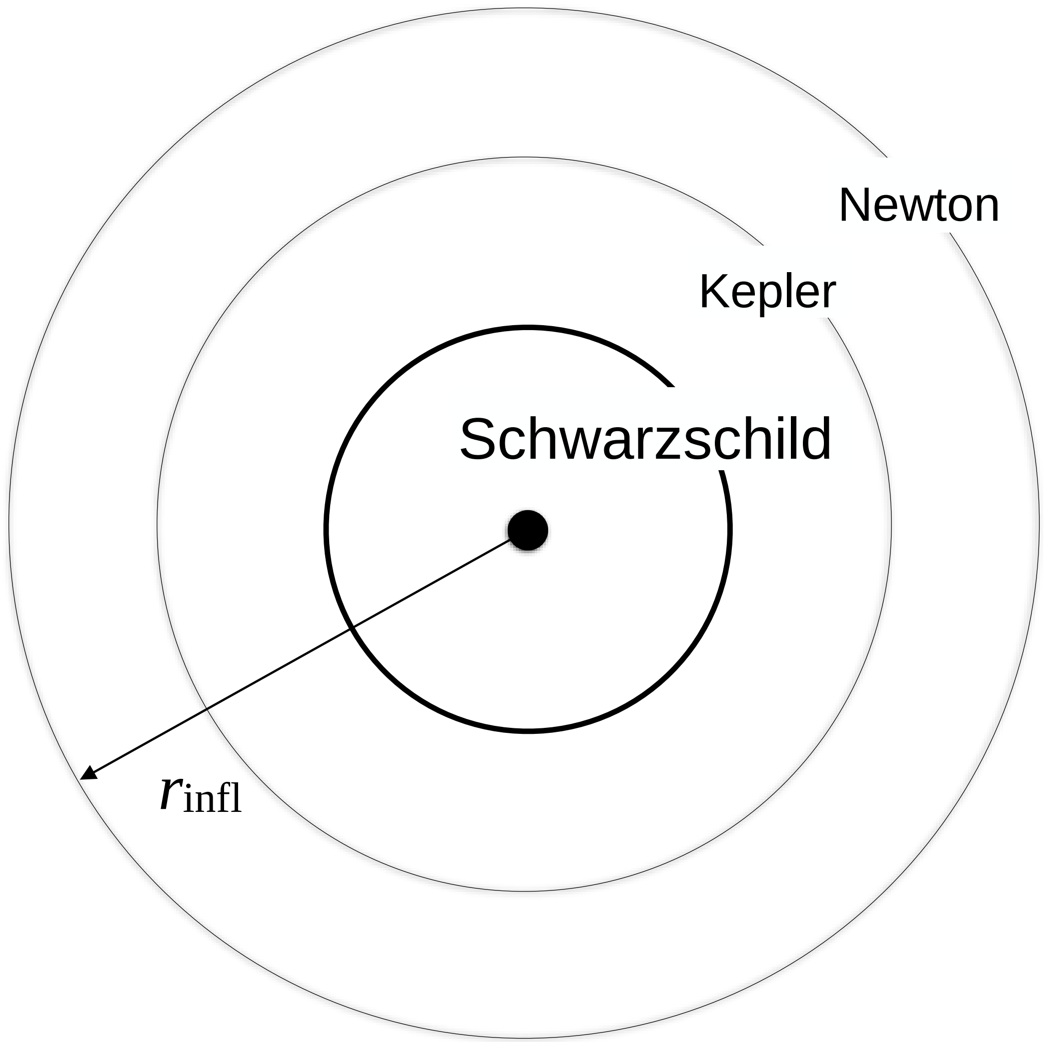
$\star$



surface density

too large

space density


Newton
Kepler
Schwarzschild
r_{infl}

Sources of Apsidal Precession

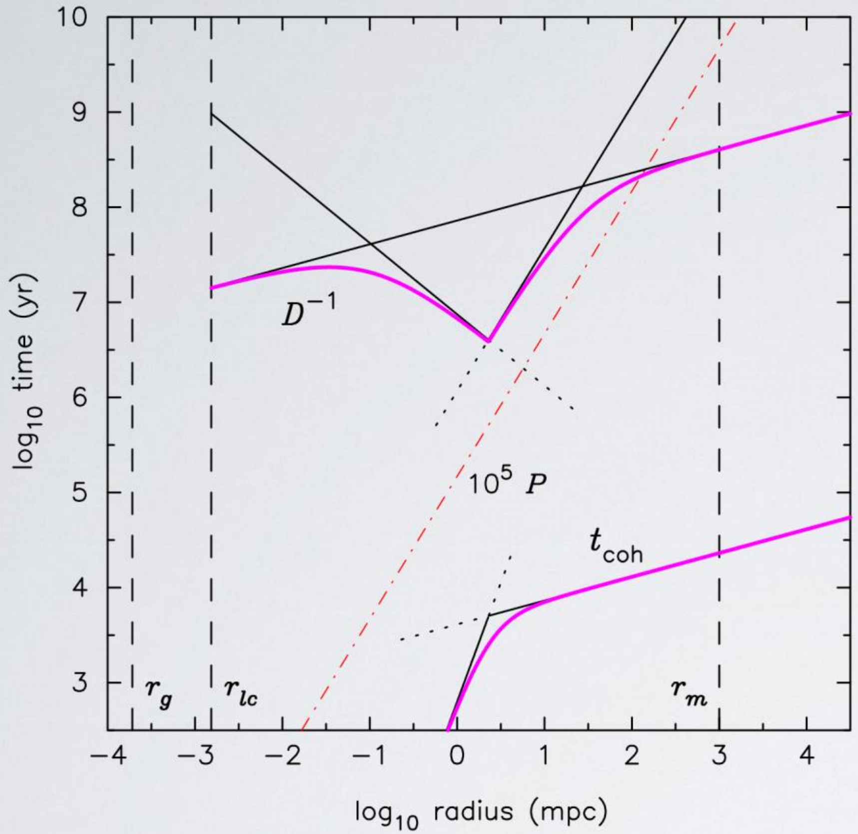
1. Relativistic ("*Schwarzschild*") precession:

$$\frac{d\omega}{dt} = \frac{2\pi}{P}\frac{3GM_\bullet}{c^2 a(1-e^2)}$$

ω = argument of periastron

2. Newtonian ("*mass*") precession:

$$\frac{d\omega}{dt} = -\frac{2\pi}{P}\frac{M_\star(a)}{M_\bullet}g(e), \qquad g(1) = 0$$



Where the coherence time is determined by GR precession.

$$a \approx \left(r_g^4\, r_{\rm infl}^5\right)^{1/9}$$

$$\frac{a}{r_{\rm infl}} \approx 1.4 \times 10^{-3} \left(\frac{M_\bullet}{4 \times 10^6 M_\odot}\right)^{4/9} \left(\frac{r_{\rm infl}}{3\ \rm pc}\right)^{-4/9}$$

"Coherence time" refers to the *average* behavior of orbits at a given radius.

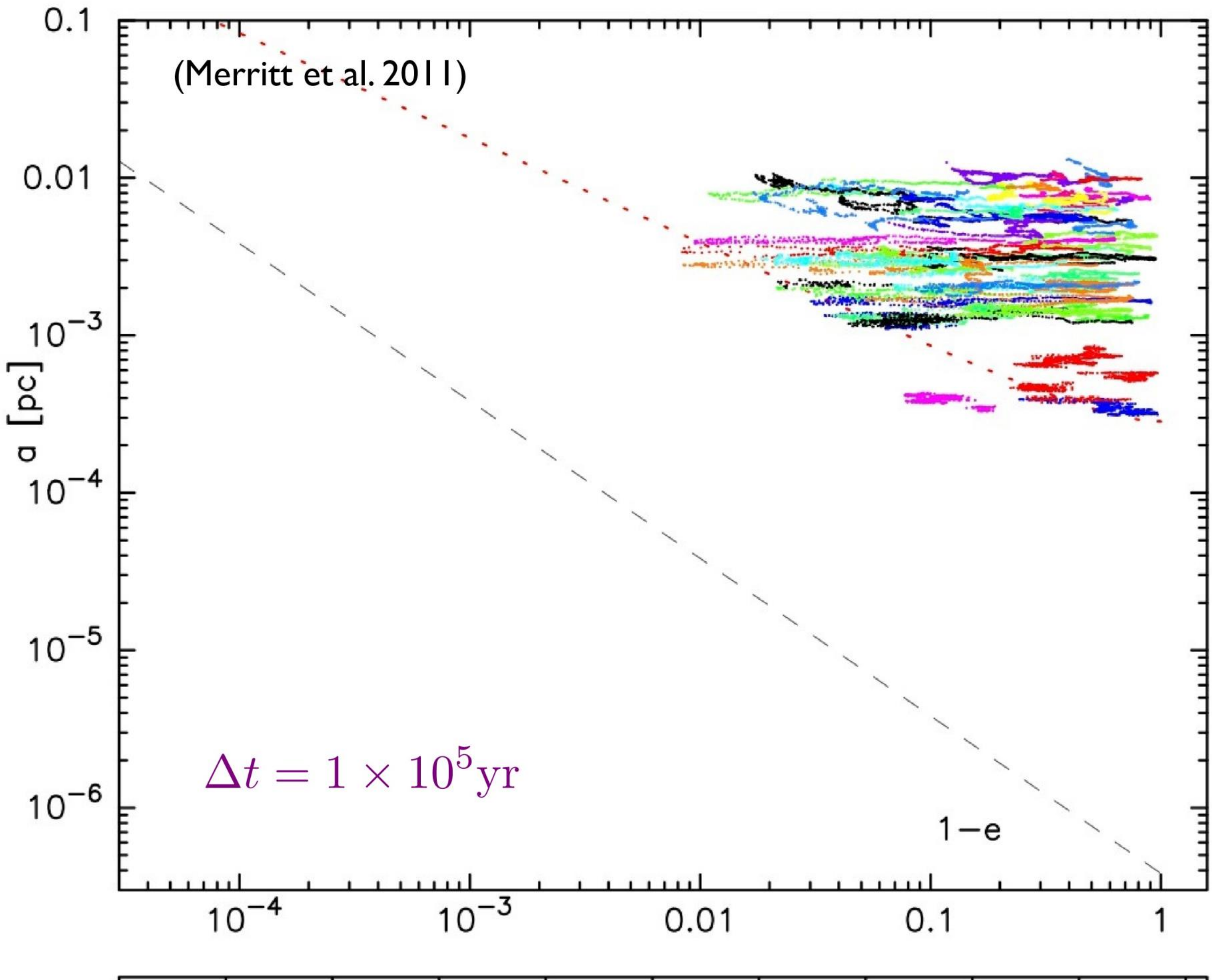
!

In the "Schwarzschild" regime, eccentric orbits can precess much faster than average.

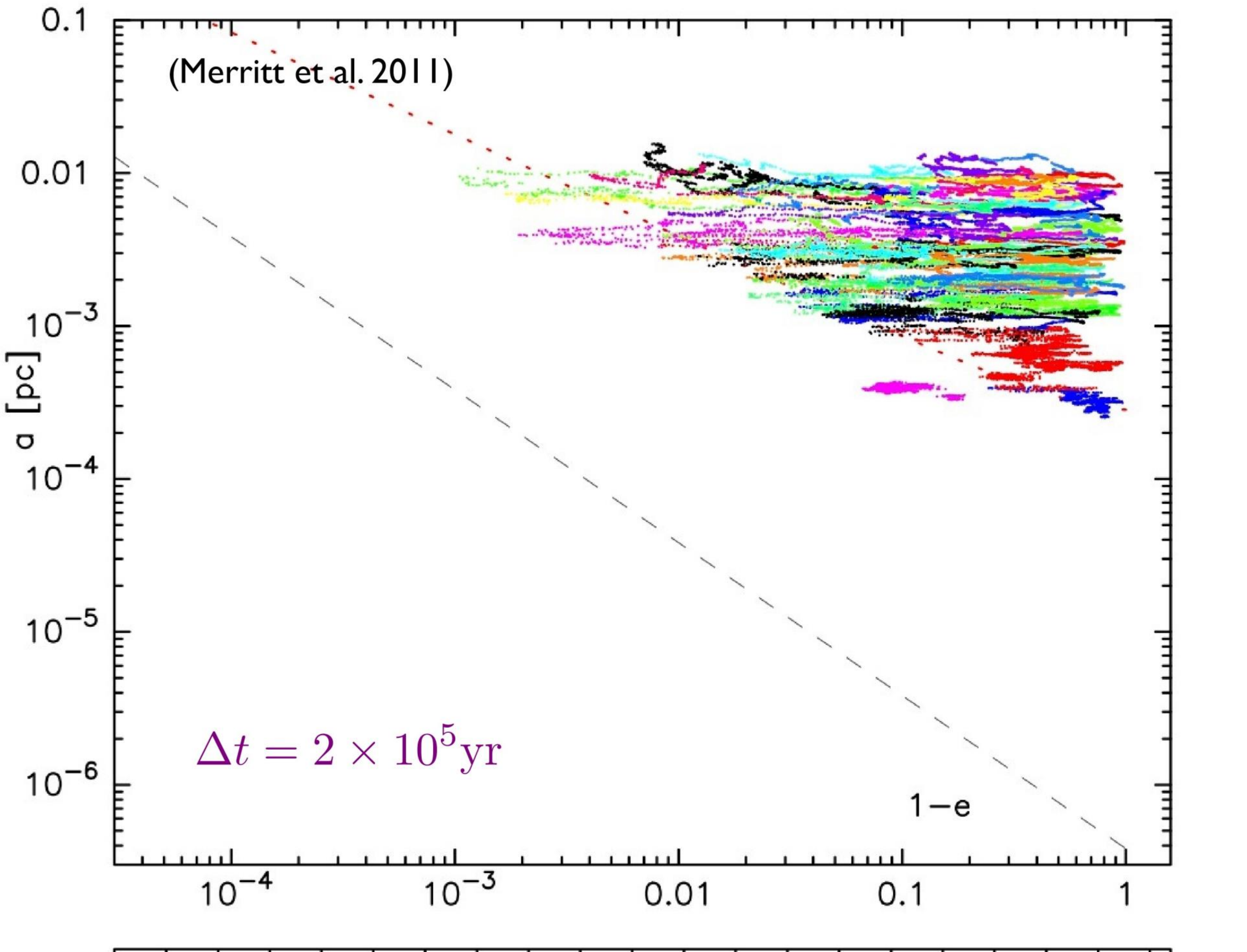
!

This is the origin of a new phenomenon: the "Schwarzschild barrier".

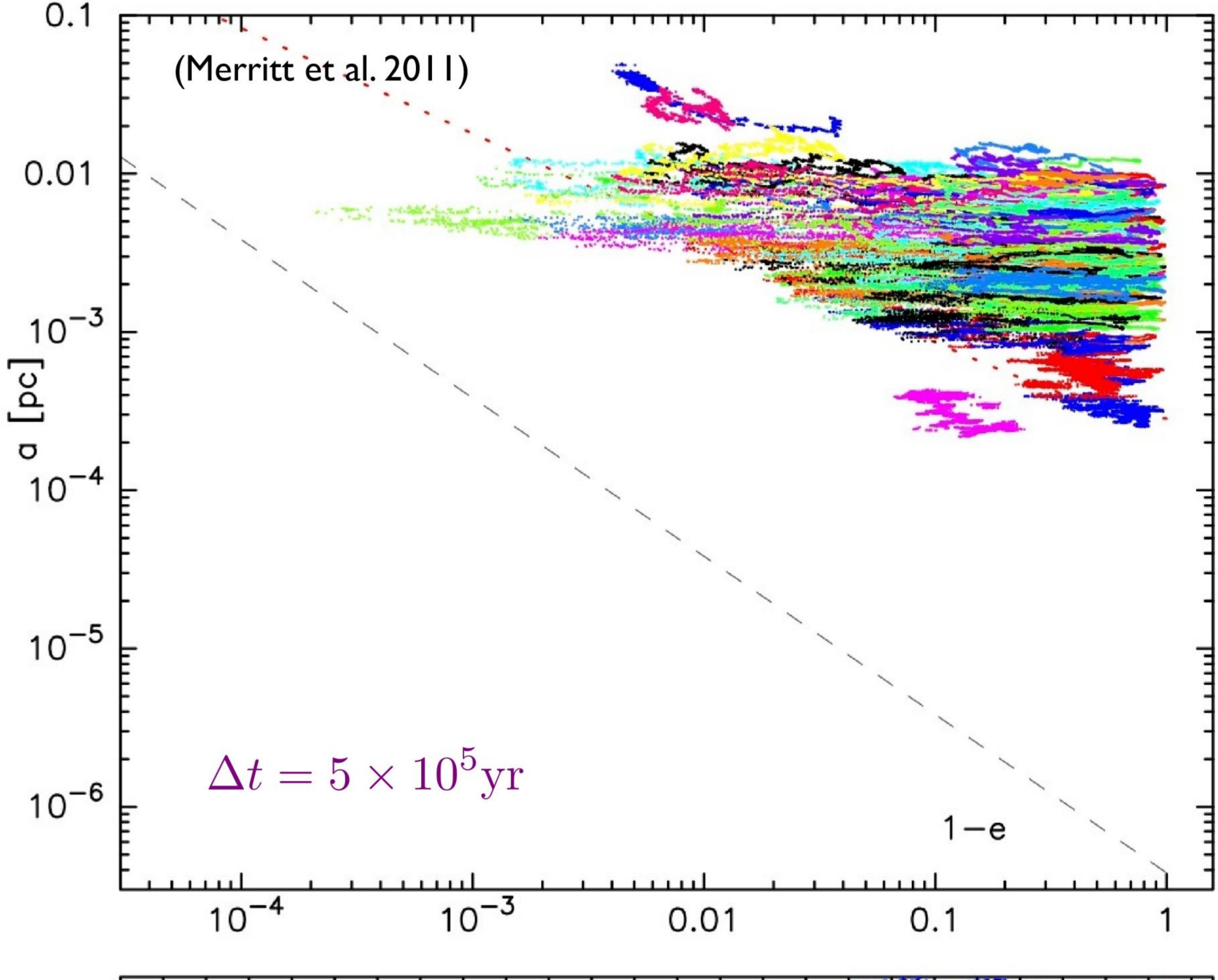
!

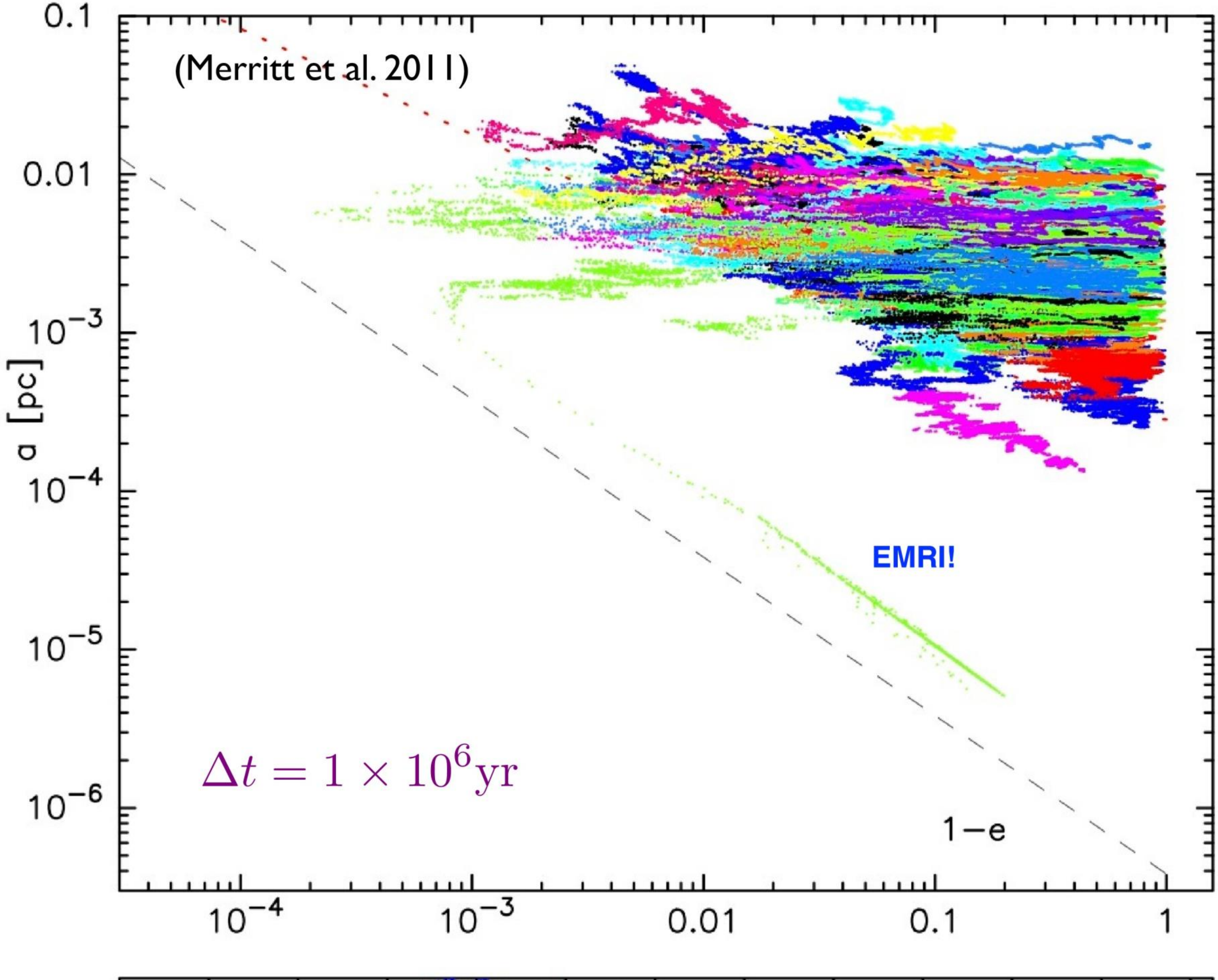


(Merritt et al. 2011)
0.1
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
a [pc]
$\Delta t = 1 \times 10^5 \mathrm{yr}$
1−e
10^{-4}
10^{-3}
0.01
0.1
1



(Merritt et al. 2011)
a [pc]
0.1
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\Delta t = 2 \times 10^5 \mathrm{yr}$
1−e
10^{-4}
10^{-3}
0.01
0.1
1


(Merritt et al. 2011)
a [pc]
0.1
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\Delta t = 5 \times 10^5 \mathrm{yr}$
1−e
10^{-4}
10^{-3}
0.01
0.1
1



(Merritt et al. 2011)
0.1
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
a [pc]
EMRI!
$\Delta t = 1 \times 10^6 \mathrm{yr}$
1−e
10^{-4}
10^{-3}
0.01
0.1
1

Two criteria have been proposed for the barrier location. Both are based on a comparison of time-scales.

Criterion I:
GR precession time = time for $\sqrt{N}$ torques to change L

$$\rightarrow \quad \sqrt{1-e^2} = \frac{r_g}{a}\frac{M_\bullet}{M_\star(a)}\sqrt{N(a)}$$

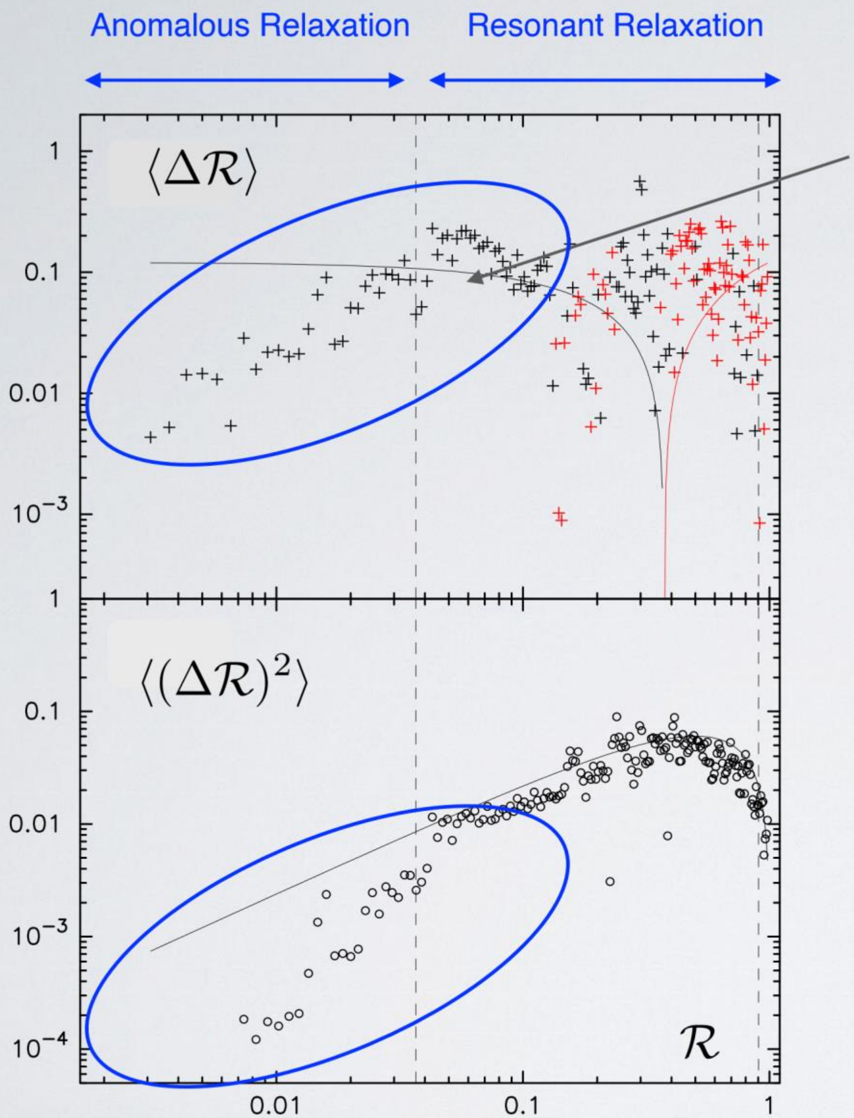
Criterion II:
GR precession time = coherence time

$$\rightarrow \quad 1-e^2 = \frac{r_g}{a}\frac{t_{\rm coh}}{P(a)}$$

(Merritt et al. 2011)

(Hamers, Portegies Zwart & Merritt 2014)

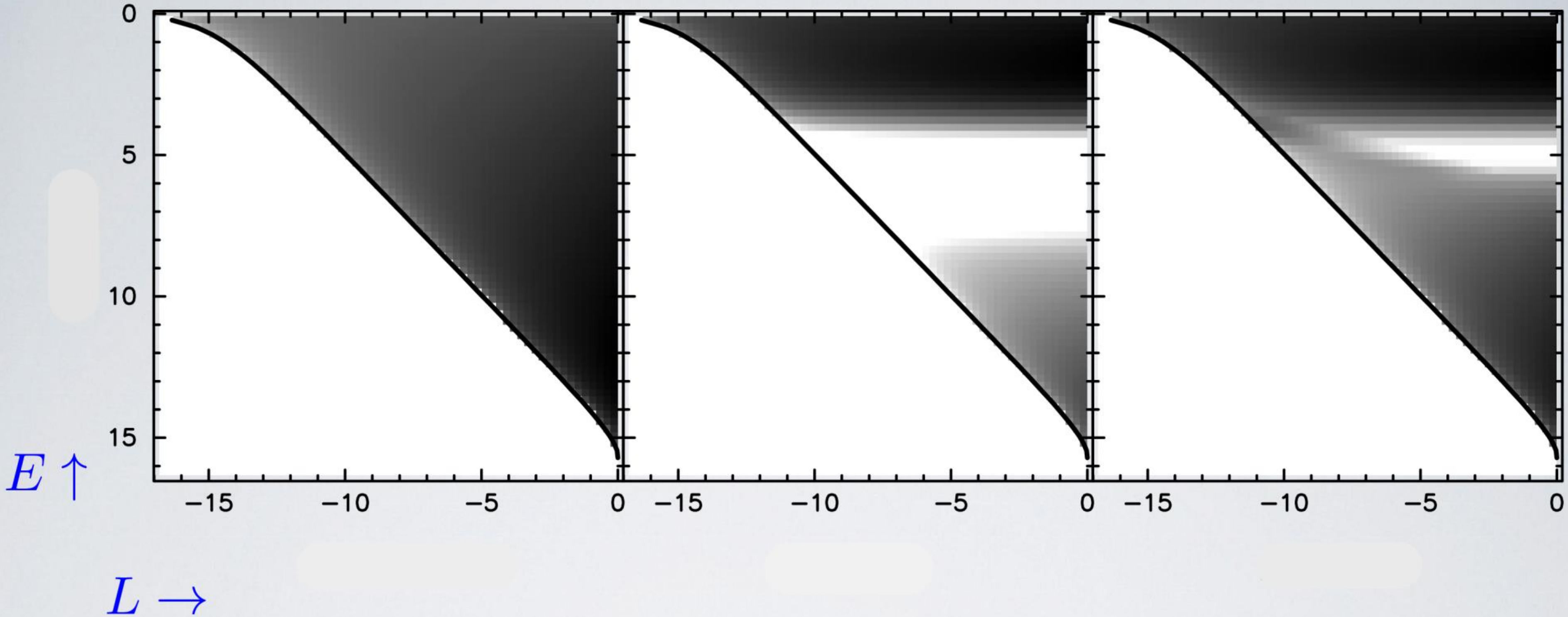
(Baror & Alexander 2014)



$$\langle \Delta\mathcal{R} \rangle = \frac{3C}{\tau}\mathcal{R}^2$$

$$\langle (\Delta\mathcal{R})^2 \rangle = \frac{4C}{\tau}\mathcal{R}^3$$

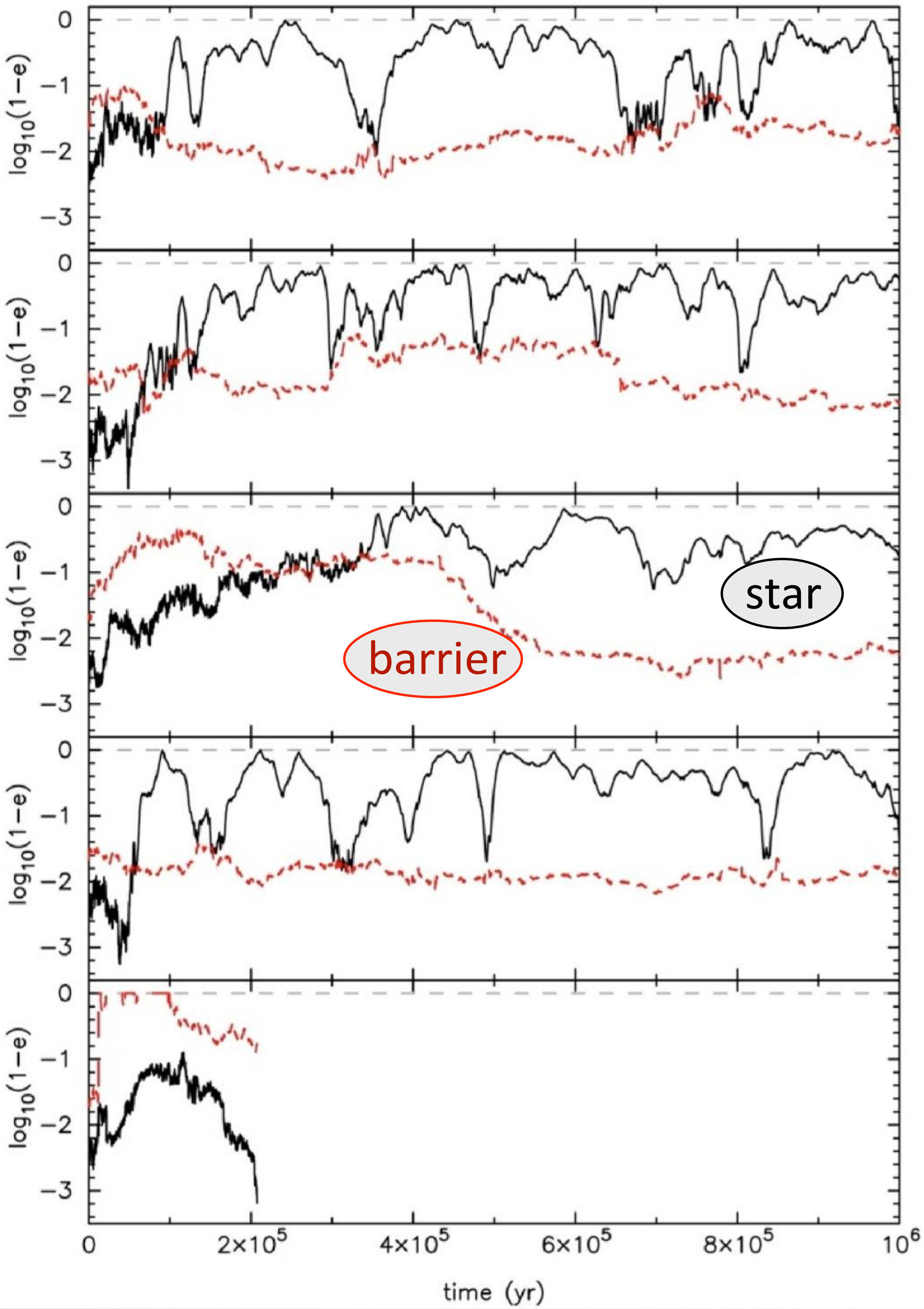
$$\tau \equiv C_2 N \left(\frac{M_\bullet}{M_\star} \frac{r_g}{a} \right)^2 t_{\mathrm{coh}}.$$



Newton

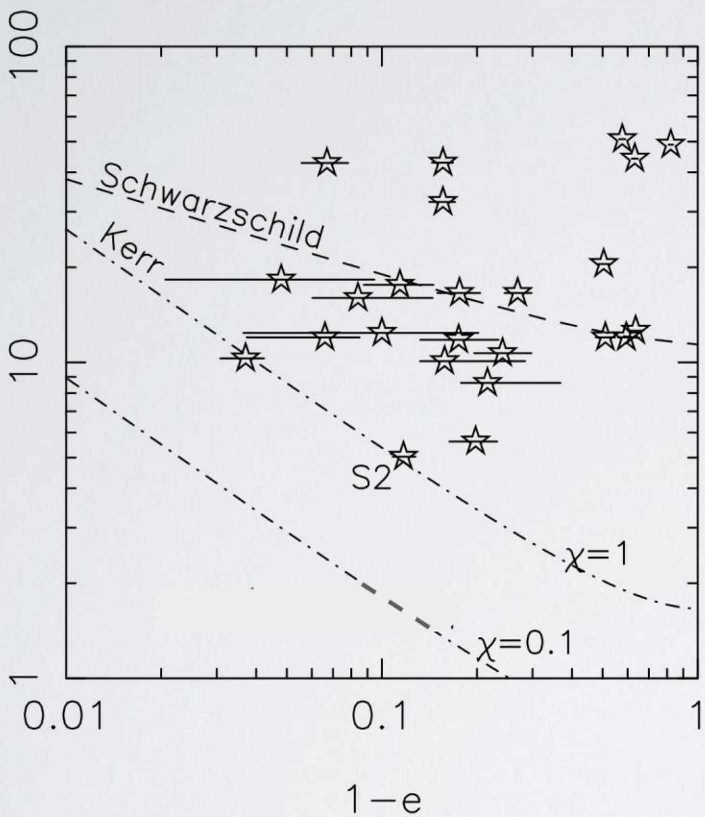
Newton
+
Kepler

Newton
+
Kepler
+
Schwarzschild



What about a star that begins its life "below the barrier"?
!
The barrier acts like a one-way membrane !

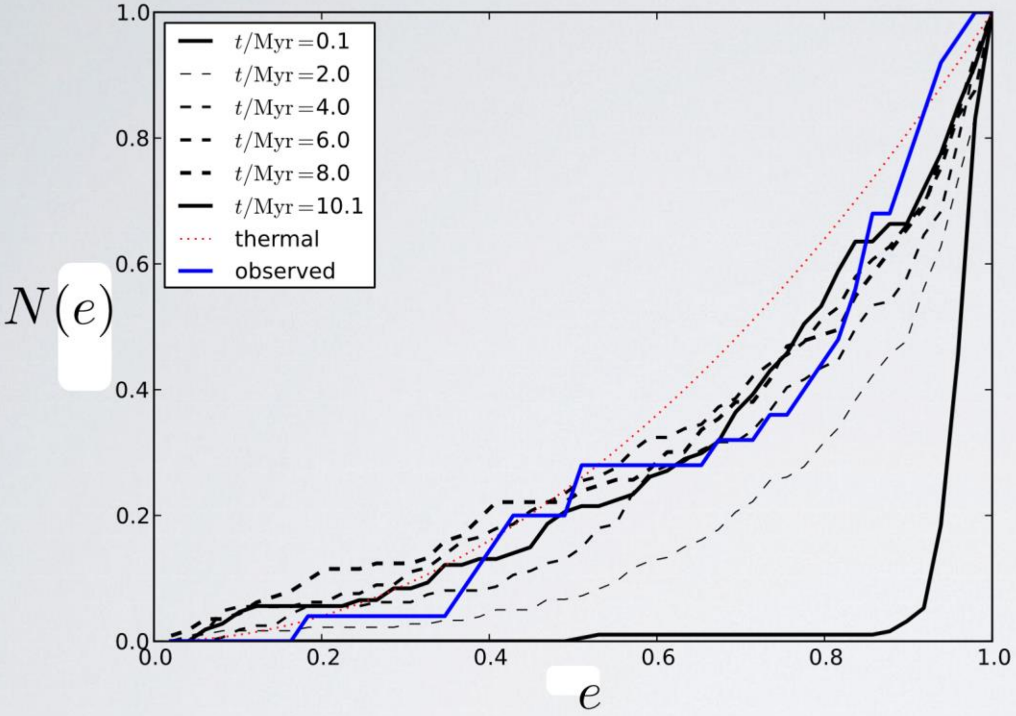
S-stars: Below the barrier ?



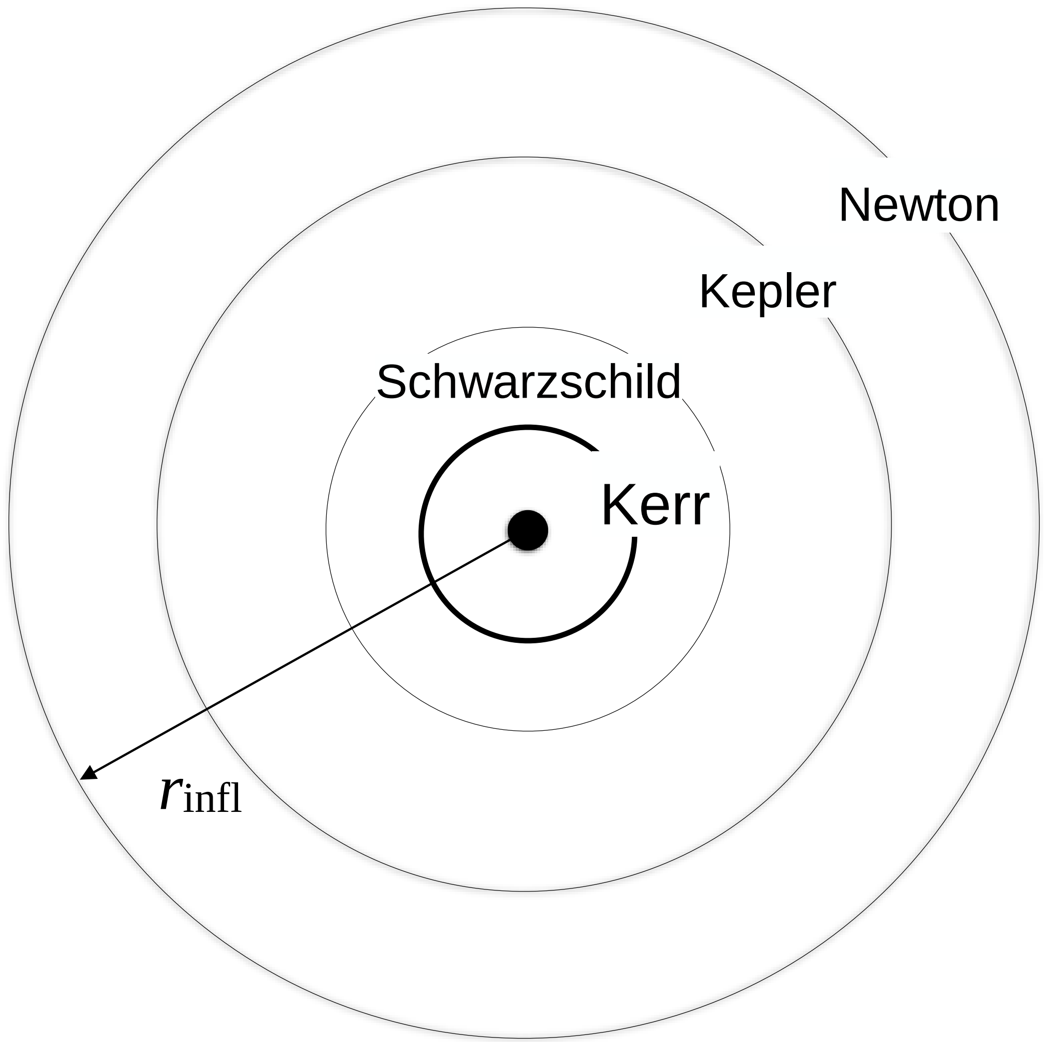
Assumes:

$$\rho_\star(r) = m_\star n(r)^{-\gamma} \quad \gamma = 7/4$$

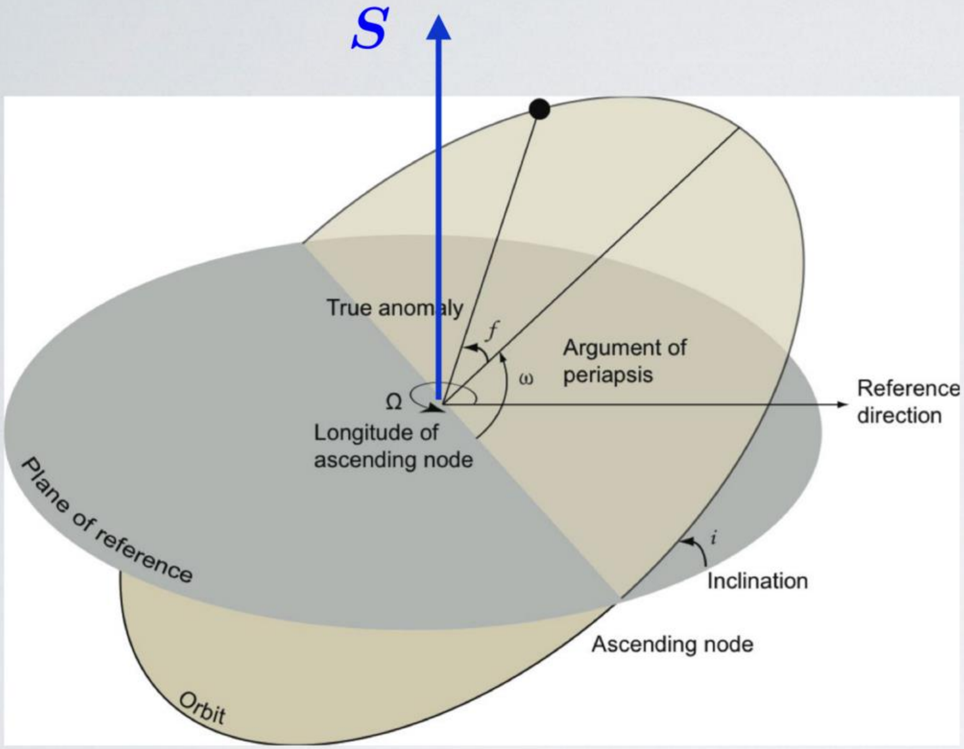
$$m_\star = 1 M_\odot$$



Evolution of the S-star eccentricity distribution.



Newton
Kepler
Schwarzschild
Kerr
r_{infl}

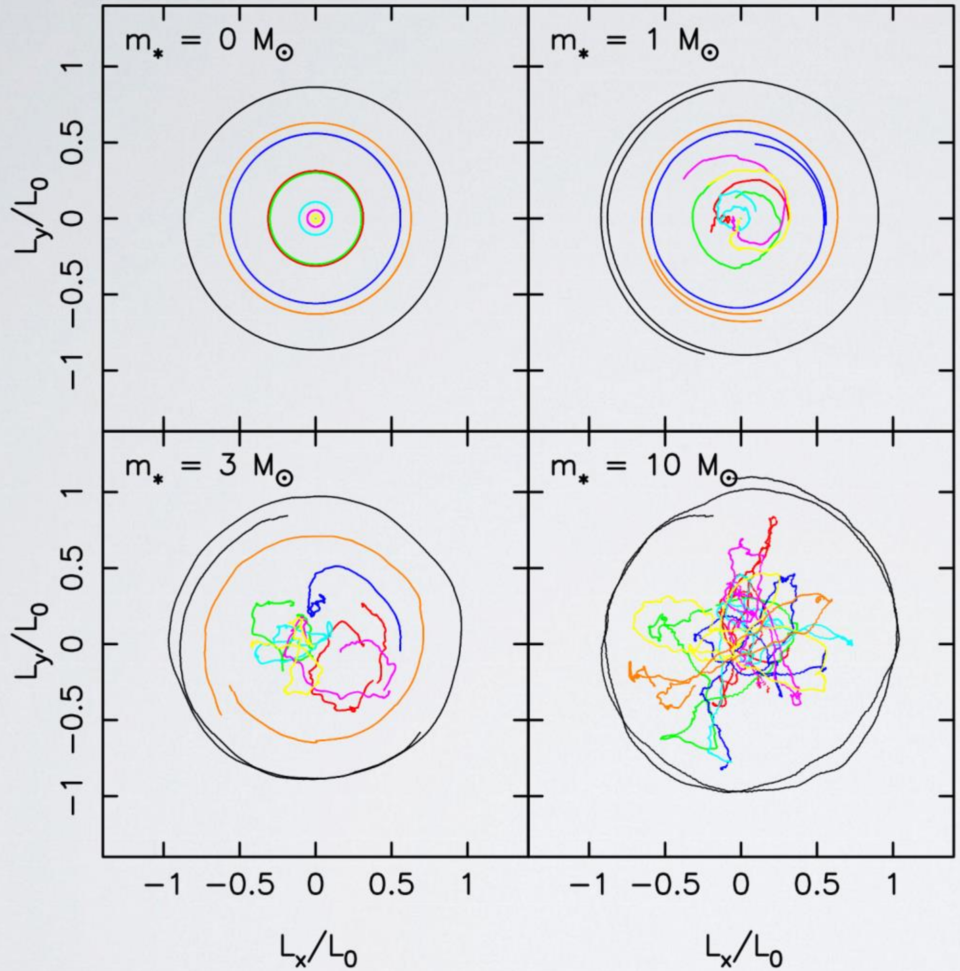


To lowest PN order, BH spin induces a nodal precession of

$$\Delta\Omega = \frac{4\pi\chi}{c^3}\left[\frac{GM_\bullet}{(1-e^2)a}\right]^{3/2}$$

per period.

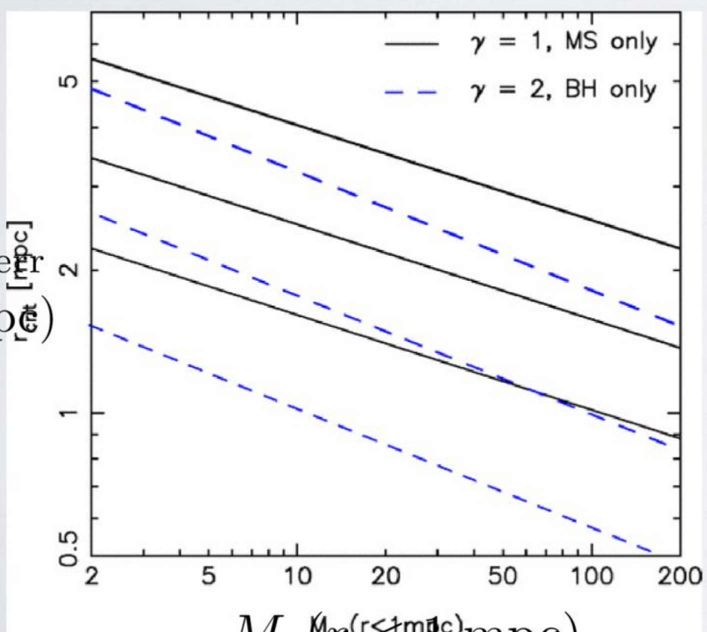
$$\boldsymbol{S} = \boldsymbol{\chi}\frac{GM_\bullet^2}{c}, \quad 0 \le |\boldsymbol{\chi}| \le 1.$$

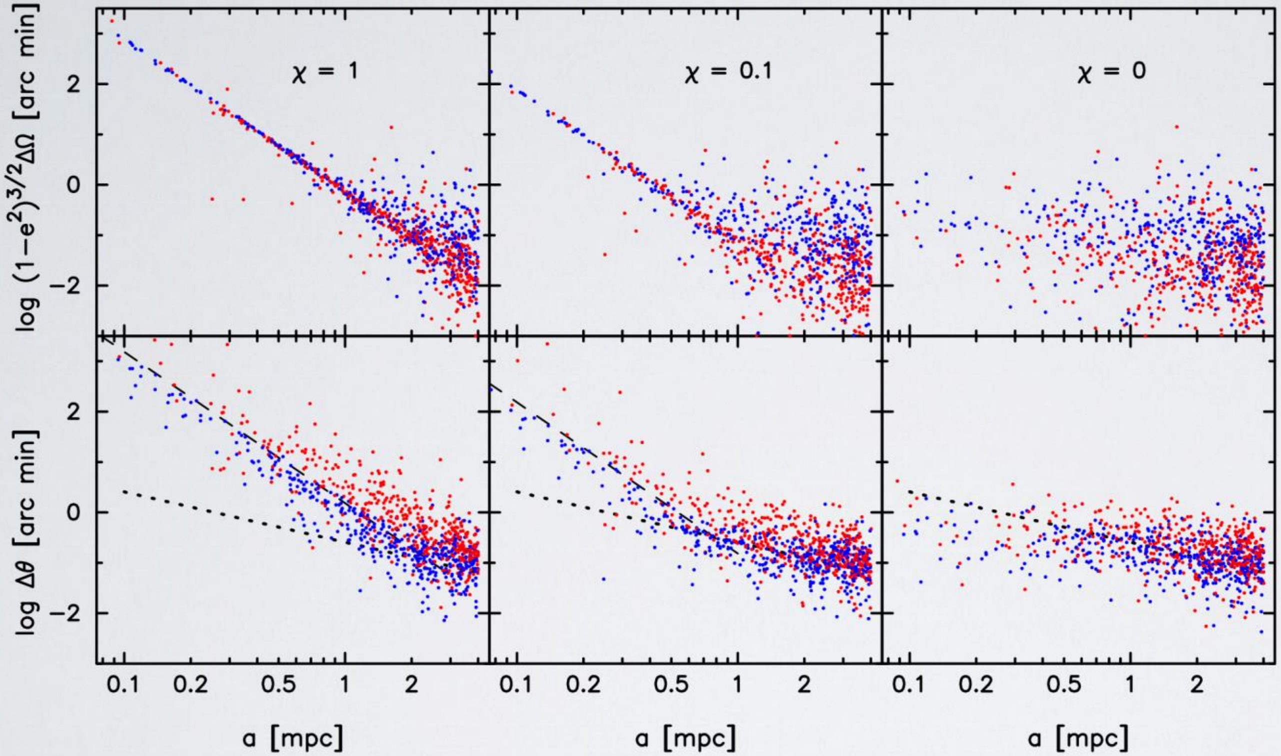


Transition from Kerr precession to Newtonian precession of orbital planes.

Lense-Thirring torques > “vector resonant relaxation” torques.

$$r_{\mathrm{Kerr}} \approx r_g \left(8\chi^2 \frac{M_\bullet}{m_\star} \right)^{1/(6-\gamma)} \left(\frac{r_m}{r_g} \right)^{(3-\gamma)/(6-\gamma)} \qquad \rho \propto r^{-\gamma}$$





$\chi = 1$
$\chi = 0.1$
$\chi = 0$
$\log\,(1-e^2)^{3/2}\Delta\Omega$ [arc min]
$\log\,\Delta\theta$ [arc min]
a [mpc]
a [mpc]
a [mpc]

Important questions:

!

— What is the steady-state distribution, $N(L)$, near the Schwarzschild Barrier?

Can it be described via the Fokker-Planck equation?

Or is a more sophisticated approach needed?

!

— How do eccentricities evolve in the “Kerr” regime?